Spring 2012

EE 330

ENGINEERING ELECTROMAGNETICS

HW 7: *Due Friday 2 March 2012*

5.2, 5.4, 5.7, 5.14, 5.21, 5.27, 5.32, 5.35, 5.37, 5.38, 6.1, 6.6

Problem 5.2 When a particle with charge q and mass m is introduced into a medium with a uniform field $\mathbf{B}$ such that the initial velocity of the particle $\mathbf{u}$ is perpendicular to $\mathbf{B}$(Fig. P5.2), the magnetic force exerted on the particle causes it to move in a circle of radius a. By equating $\mathbf{F}_\mathrm{m}$ to the centripetal force on the particle, determine a in terms of q, m, u, and $\mathbf{B}$.

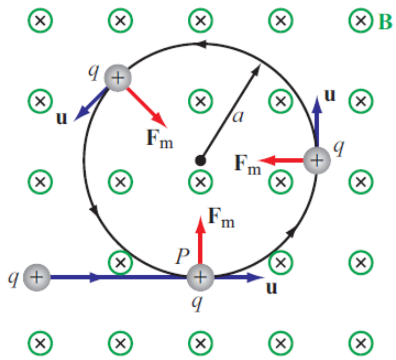

Figure P5.2: Particle of charge q projected with velocity $\mathbf{u}$ into a medium with a uniform field $\mathbf{B}$ perpendicular to $\mathbf{u}$ (Problem 5.2).

Solution: The centripetal force acting on the particle is given by $F_\mathrm{c} = mu^2/a$. Equating F_c to F_m given by Eq. (5.4), we have $mu^2/a = quB\sin\theta$. Since the magnetic field is perpendicular to the particle velocity, $\sin\theta = 1$. Hence, $a = mu/qB$.

Problem 5.4 The rectangular loop shown in Fig. P5.4 consists of 20 closely wrapped turns and is hinged along the z-axis. The plane of the loop makes an angle of 30° with the y-axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform field $\mathbf{B} = \hat{\mathbf{y}}2.4$ T? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?

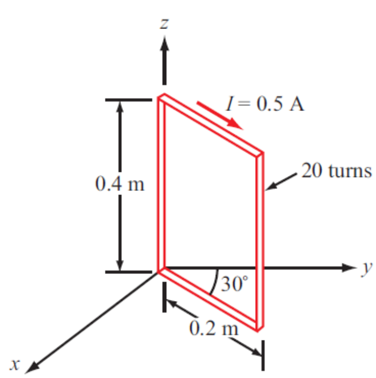

Figure P5.4: Hinged rectangular loop of Problem 5.4.

Solution: The magnetic torque on a loop is given by $\mathbf{T} = \mathbf{m} \times \mathbf{B}$ (Eq. (5.20)), where $\mathbf{m} = \hat{\mathbf{n}}NIA$ (Eq. (5.19)). For this problem, it is given that $I = 0.5$ A, $N = 20$ turns, and $A = 0.2\text{ m} \times 0.4\text{ m} = 0.08\text{ m}^2$. From the figure, $\hat{\mathbf{n}} = -\hat{\mathbf{x}}\cos 30° + \hat{\mathbf{y}}\sin 30°$. Therefore, $\mathbf{m} = \hat{\mathbf{n}}0.8\ (\mathrm{A}\cdot\mathrm{m}^2)$ and $\mathbf{T} = \hat{\mathbf{n}}0.8\ (\mathrm{A}\cdot\mathrm{m}^2) \times \hat{\mathbf{y}}2.4\text{ T} = -\hat{\mathbf{z}}1.66$ (N·m). As the torque is negative, the direction of rotation is clockwise, looking from above.

Problem 5.7 An 8 cm $\times$ 12 cm rectangular loop of wire is situated in the x–y plane with the center of the loop at the origin and its long sides parallel to the x-axis. The loop has a current of 50 A flowing clockwise (when viewed from above). Determine the magnetic field at the center of the loop.

Solution: The total magnetic field is the vector sum of the individual fields of each of the four wire segments: $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4$. An expression for the magnetic field from a wire segment is given by Eq. (5.29). For all segments shown in Fig. P5.7, the

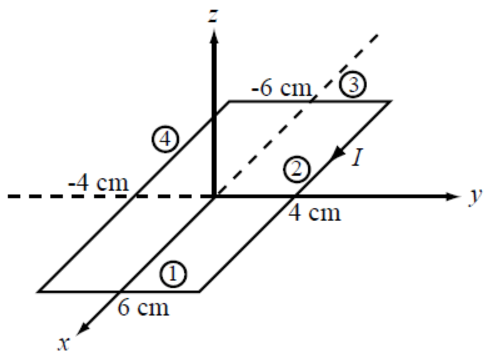

Figure P5.7: Problem 5.7.

combination of the direction of the current and the right-hand rule gives the direction of the magnetic field as $-z$ direction at the origin. With $r = 6$ cm and $l = 8$ cm,

$$\mathbf{B}_1 = -\hat{\mathbf{z}}\frac{\mu I l}{2\pi r\sqrt{4r^2 + l^2}}$$
$$= -\hat{\mathbf{z}}\frac{4\pi \times 10^{-7} \times 50 \times 0.08}{2\pi \times 0.06 \times \sqrt{4 \times 0.06^2 + 0.08^2}} = -\hat{\mathbf{z}}9.24 \times 10^{-5} \quad \text{(T)}.$$

For segment 2, $r = 4$ cm and $l = 12$ cm,

$$\mathbf{B}_2 = -\hat{\mathbf{z}}\frac{\mu I l}{2\pi r\sqrt{4r^2 + l^2}}$$
$$= -\hat{\mathbf{z}}\frac{4\pi \times 10^{-7} \times 50 \times 0.12}{2\pi \times 0.04 \times \sqrt{4 \times 0.04^2 + 0.12^2}} = -\hat{\mathbf{z}}20.80 \times 10^{-5} \quad \text{(T)}.$$

Similarly,

$$\mathbf{B}_3 = -\hat{\mathbf{z}}9.24 \times 10^{-5} \quad \text{(T)}, \qquad \mathbf{B}_4 = -\hat{\mathbf{z}}20.80 \times 10^{-5} \quad \text{(T)}.$$

The total field is then $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4 = -\hat{\mathbf{z}}0.60$ (mT).

Problem 5.14 Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the x–y plane with its center at the origin, and the second loop's center is at $z = 2$ m. If the two loops have the same radius $a = 3$ m, determine the magnetic field at:

(a) $z = 0$

(b) $z = 1$ m

(c) $z = 2$ m

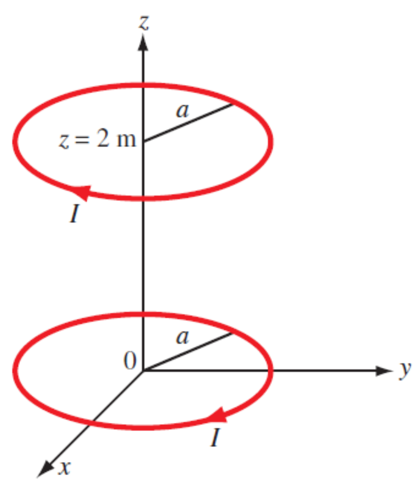

Figure P5.14: Parallel circular loops of Problem 5.14.

Solution: The magnetic field due to a circular loop is given by (5.34) for a loop in the x–y plane carrying a current I in the $+\hat{\boldsymbol{\phi}}$-direction. Considering that the bottom loop in Fig. is in the x–y plane, but the current direction is along $-\hat{\boldsymbol{\phi}}$,

$$\mathbf{H}_1 = -\hat{\mathbf{z}}\,\frac{Ia^2}{2(a^2+z^2)^{3/2}},$$

where z is the observation point along the z-axis. For the second loop, which is at a height of 2 m, we can use the same expression but z should be replaced with $(z-2)$. Hence,

$$\mathbf{H}_2 = -\hat{\mathbf{z}}\,\frac{Ia^2}{2[a^2+(z-2)^2]^{3/2}}.$$

The total field is

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 = -\hat{\mathbf{z}}\,\frac{Ia^2}{2}\left[\frac{1}{(a^2+z^2)^{3/2}} + \frac{1}{[a^2+(z-2)^2]^{3/2}}\right] \text{ A/m.}$$

(a) At $z = 0$, and with $a = 3$ m and $I = 40$ A,

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40\times 9}{2}\left[\frac{1}{3^3} + \frac{1}{(9+4)^{3/2}}\right] = -\hat{\mathbf{z}}\,10.5 \text{ A/m.}$$

(b) At $z = 1$ m (midway between the loops):

$$\mathbf{H} = -\hat{\mathbf{z}}\,\frac{40\times 9}{2}\left[\frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}}\right] = -\hat{\mathbf{z}}\,11.38 \text{ A/m.}$$

(c) At $z = 2$ m, $\mathbf{H}$ should be the same as at $z = 0$. Thus,

$$\mathbf{H} = -\hat{\mathbf{z}}\,10.5 \text{ A/m.}$$

Problem 5.21 Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius a, and the inner and outer radii of the outer conductor are b and c, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \leq r \leq a$, $a \leq r \leq b$, $b \leq r \leq c$, and $r \geq c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of r over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

Solution:

(a) Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi(c^2 - b^2)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius r is therefore

$$I_{\text{enclosed}} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I\frac{c^2 - r^2}{c^2 - b^2},$$

and the resulting magnetic field is

$$\mathbf{H} = \hat{\boldsymbol{\phi}} \frac{I_{\text{enclosed}}}{2\pi r} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

For $r > c$, the total enclosed current is zero: the total current flowing on the inner conductor is equal to the total current flowing on the outer conductor, but they are flowing in opposite directions. Therefore, $\mathbf{H} = 0$.

(b) See Fig. P5.21.

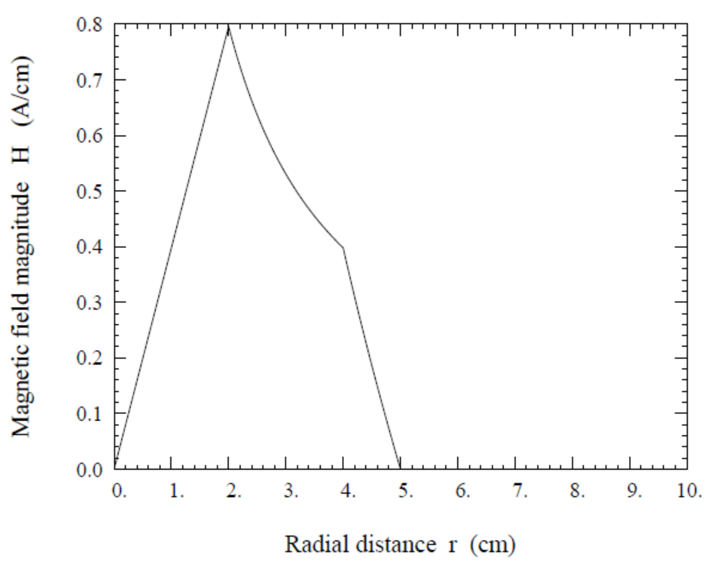

Figure P5.21: Problem 5.21.

Problem 5.27 In a given region of space, the vector magnetic potential is given by $\mathbf{A} = \hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x)$ (Wb/m).

(a) Determine $\mathbf{B}$.

(b) Use Eq. (5.66) to calculate the magnetic flux passing through a square loop with 0.25-m-long edges if the loop is in the x–y plane, its center is at the origin, and its edges are parallel to the x- and y-axes.

(c) Calculate Φ again using Eq. (5.67).

Solution:

(a) From Eq. (5.53), $\mathbf{B} = \nabla\times\mathbf{A} = \hat{\mathbf{z}}5\pi\sin\pi y - \hat{\mathbf{y}}\pi\cos\pi x$.

(b) From Eq. (5.66),

$$\begin{aligned}\Phi = \iint \mathbf{B}\cdot d\mathbf{s} &= \int_{y=-0.125\text{ m}}^{0.125\text{ m}}\int_{x=-0.125\text{ m}}^{0.125\text{ m}} (\hat{\mathbf{z}}5\pi\sin\pi y - \hat{\mathbf{y}}\pi\cos\pi x)\cdot(\hat{\mathbf{z}}\,dx\,dy)\\ &= \left(\left(-5\pi x\frac{\cos\pi y}{\pi}\right)\Bigg|_{x=-0.125}^{0.125}\right)\Bigg|_{y=-0.125}^{0.125}\\ &= \frac{-5}{4}\left(\cos\left(\frac{\pi}{8}\right)-\cos\left(\frac{-\pi}{8}\right)\right) = 0.\end{aligned}$$

(c) From Eq. (5.67), $\Phi = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$, where C is the square loop in the x-y plane with sides of length 0.25 m centered at the origin. Thus, the integral can be written as

$$\Phi = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}},$$

where S_{front}, S_{back}, S_{left}, and S_{right} are the sides of the loop.

$$\begin{aligned}S_{\text{front}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))\big|_{y=-0.125}\cdot(\hat{\mathbf{x}}\,dx)\\ &= \int_{x=-0.125}^{0.125} 5\cos\pi y\big|_{y=-0.125}\,dx\\ &= \left((5x\cos\pi y)\big|_{y=-0.125}\right)\Big|_{x=-0.125}^{0.125} = \frac{5}{4}\cos\left(\frac{-\pi}{8}\right) = \frac{5}{4}\cos\left(\frac{\pi}{8}\right),\\ S_{\text{back}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))\big|_{y=0.125}\cdot(-\hat{\mathbf{x}}\,dx)\\ &= -\int_{x=-0.125}^{0.125} 5\cos\pi y\big|_{y=0.125}\,dx\\ &= \left((-5x\cos\pi y)\big|_{y=0.125}\right)\Big|_{x=-0.125}^{0.125} = -\frac{5}{4}\cos\left(\frac{\pi}{8}\right),\end{aligned}$$

$$\begin{aligned}S_{\text{left}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))\big|_{x=-0.125}\cdot(-\hat{\mathbf{y}}\,dy)\\ &= -\int_{y=-0.125}^{0.125} 0\big|_{x=-0.125}\,dy = 0,\\ S_{\text{right}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5\cos\pi y + \hat{\mathbf{z}}(2+\sin\pi x))\big|_{x=0.125}\cdot(\hat{\mathbf{y}}\,dy)\\ &= \int_{y=-0.125}^{0.125} 0\big|_{x=0.125}\,dy = 0.\end{aligned}$$

Thus,

$$\Phi = \oint_c \mathbf{A}\cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}} = \frac{5}{4}\cos\left(\frac{\pi}{8}\right) - \frac{5}{4}\cos\left(\frac{\pi}{8}\right) + 0 + 0 = 0.$$

Problem 5.32 The x–y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2 (Fig. P5.32). If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} H_{1z}$$

find:

(a) $\mathbf{H}_2$

(b) θ_1 and θ_2

(c) Evaluate $\mathbf{H}_2$, θ_1, and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$

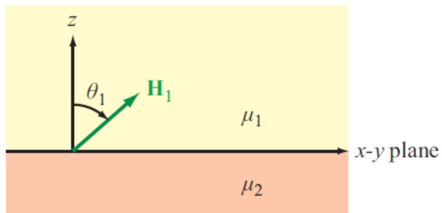

Figure P5.32: Adjacent magnetic media (Problem 5.32).

Solution:

(a) From (5.80),

$$\mu_1 H_{1n} = \mu_2 H_{2n},$$

and in the absence of surface currents at the interface, (5.85) states

$$H_{1t} = H_{2t}.$$

In this case, $H_{1z} = H_{1n}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$
$$H_{1x} = H_{2x},$$
$$H_{1y} = H_{2y},$$

and

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} \frac{\mu_1}{\mu_2} H_{1z}.$$

(b)

$$H_{1t} = \sqrt{H_{1x}^2 + H_{1y}^2},$$

$$\tan\theta_1 = \frac{H_{1t}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{H_{1z}},$$

$$\tan\theta_2 = \frac{H_{2t}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1} \tan\theta_1.$$

(c)

$$\mathbf{H}_2 = \hat{\mathbf{x}}\, 2 + \hat{\mathbf{z}} \frac{1}{4} \cdot 4 = \hat{\mathbf{x}}\, 2 + \hat{\mathbf{z}} \quad \text{(A/m)},$$

$$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56°,$$

$$\theta_2 = \tan^{-1}\left(\frac{2}{1}\right) = 63.44°.$$

Problem 5.35 The plane boundary defined by $z = 0$ separates air from a block of iron. If $\mathbf{B}_1 = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8$ in air ($z \geq 0$), find $\mathbf{B}_2$ in iron ($z \leq 0$), given that $\mu = 5000\mu_0$ for iron.

Solution: From Eq. (5.2),

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{\mu_1}(\hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8).$$

The z component is the normal component to the boundary at $z = 0$. Therefore, from Eq. (5.79), $B_{2z} = B_{1z} = 8$ while, from Eq. (5.85),

$$H_{2x} = H_{1x} = \frac{1}{\mu_1}4, \qquad H_{2y} = H_{1y} = -\frac{1}{\mu_1}6,$$

or

$$B_{2x} = \mu_2 H_{2x} = \frac{\mu_2}{\mu_1}4, \qquad B_{2y} = \mu_2 H_{2y} = -\frac{\mu_2}{\mu_1}6,$$

where $\mu_2/\mu_1 = \mu_r = 5000$. Therefore,

$$\mathbf{B}_2 = \hat{\mathbf{x}}20000 - \hat{\mathbf{y}}30000 + \hat{\mathbf{z}}8.$$

Problem 5.37 Obtain an expression for the self-inductance per unit length for the parallel wire transmission line of Fig. 5-27(a) in terms of a, d, and μ, where a is the radius of the wires, d is the axis-to-axis distance between the wires, and μ is the permeability of the medium in which they reside.

Solution:

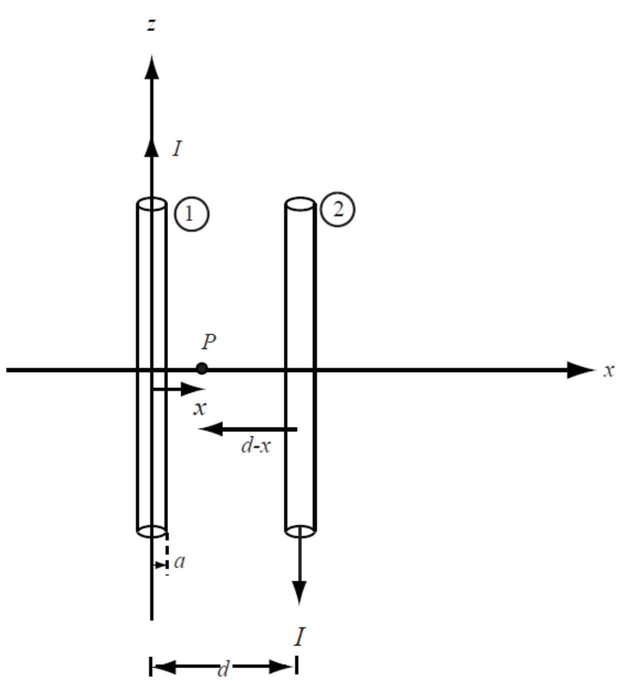

Figure P5.37: Parallel wire transmission line.

Let us place the two wires in the x–z plane and orient the current in one of them to be along the $+z$-direction and the current in the other one to be along the $-z$-direction, as shown in Fig. P5.37. From Eq. (5.30), the magnetic field at point $P = (x, 0, z)$ due to wire 1 is

$$\mathbf{B}_1 = \hat{\boldsymbol{\phi}}\frac{\mu I}{2\pi r} = \hat{\mathbf{y}}\frac{\mu I}{2\pi x},$$

where the permeability has been generalized from free space to any substance with permeability μ, and it has been recognized that in the x-z plane, $\hat{\boldsymbol{\phi}} = \hat{\mathbf{y}}$ and $r = x$ as long as $x > 0$.

Given that the current in wire 2 is opposite that in wire 1, the magnetic field created by wire 2 at point $P = (x, 0, z)$ is in the same direction as that created by wire 1, and it is given by

$$\mathbf{B}_2 = \hat{\mathbf{y}}\frac{\mu I}{2\pi(d-x)}.$$

Therefore, the total magnetic field in the region between the wires is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \hat{\mathbf{y}}\frac{\mu I}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right) = \hat{\mathbf{y}}\frac{\mu I d}{2\pi x(d-x)}.$$

From Eq. (5.91), the flux crossing the surface area between the wires over a length l of the wire structure is

$$\begin{aligned}
\Phi = \iint_S \mathbf{B}\cdot d\mathbf{s} &= \int_{z=z_0}^{z_0+l}\int_{x=a}^{d-a}\left(\hat{\mathbf{y}}\frac{\mu I d}{2\pi x(d-x)}\right)\cdot(\hat{\mathbf{y}}\,dx\,dz)\\
&= \frac{\mu I l d}{2\pi}\left(\frac{1}{d}\ln\left(\frac{x}{d-x}\right)\right)\Bigg|_{x=a}^{d-a}\\
&= \frac{\mu I l}{2\pi}\left(\ln\left(\frac{d-a}{a}\right) - \ln\left(\frac{a}{d-a}\right)\right)\\
&= \frac{\mu I l}{2\pi}\times 2\ln\left(\frac{d-a}{a}\right) = \frac{\mu I l}{\pi}\ln\left(\frac{d-a}{a}\right).
\end{aligned}$$

Since the number of 'turns' in this structure is 1, Eq. (5.93) states that the flux linkage is the same as magnetic flux: $\Lambda = \Phi$. Then Eq. (5.94) gives a total inductance over the length l as

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{\mu l}{\pi}\ln\left(\frac{d-a}{a}\right) \quad \text{(H)}.$$

Therefore, the inductance per unit length is

$$L' = \frac{L}{l} = \frac{\mu}{\pi}\ln\left(\frac{d-a}{a}\right) \approx \frac{\mu}{\pi}\ln\left(\frac{d}{a}\right) \quad \text{(H/m)},$$

where the last approximation recognizes that the wires are thin compared to the separation distance (i.e., that $d \gg a$). This has been an implied condition from the beginning of this analysis, where the flux passing through the wires themselves have been ignored. This is the thin-wire limit in Table 2-1 for the two wire line.

Problem 5.38 A solenoid with a length of 20 cm and a radius of 5 cm consists of 400 turns and carries a current of 12 A. If $z = 0$ represents the midpoint of the solenoid, generate a plot for $|\mathbf{H}(z)|$ as a function of z along the axis of the solenoid for the range -20 cm $\leq z \leq 20$ cm in 1-cm steps.

Solution:

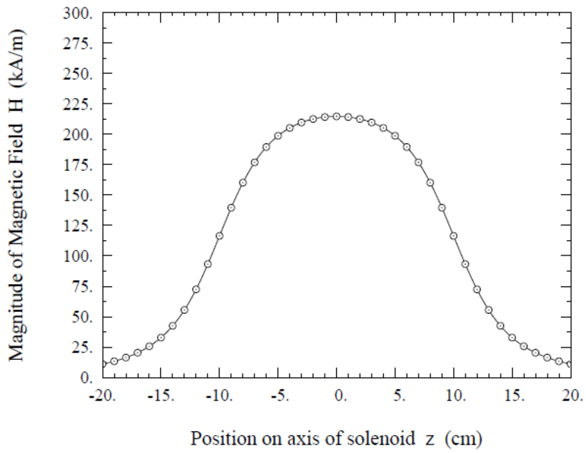

Figure P5.38: Problem 5.38.

Let the length of the solenoid be $l = 20$ cm. From Eq. (5.88a) and Eq. (5.88b), $z = a\tan\theta$ and $a^2 + t^2 = a^2\sec^2\theta$, which implies that $z/\sqrt{z^2+a^2} = \sin\theta$. Generalizing this to an arbitrary observation point z' on the axis of the solenoid, $(z-z')/\sqrt{(z-z')^2+a^2} = \sin\theta$. Using this in Eq. (5.89),

$$\begin{aligned}
\mathbf{H}(0,0,z') = \frac{\mathbf{B}}{\mu} &= \hat{\mathbf{z}}\frac{nI}{2}(\sin\theta_2 - \sin\theta_1) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} - \frac{-l/2-z'}{\sqrt{(-l/2-z')^2+a^2}}\right) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} + \frac{l/2+z'}{\sqrt{(l/2+z')^2+a^2}}\right) \quad \text{(A/m)}.
\end{aligned}$$

A plot of the magnitude of this function of z' with $a = 5$ cm, $n = 400$ turns/20 cm = 20,000 turns/m, and $I = 12$ A appears in Fig. P5.38.

Problem 6.1 The switch in the bottom loop of Fig. P6.1 is closed at $t = 0$ and then opened at a later time t_1. What is the direction of the current I in the top loop (clockwise or counterclockwise) at each of these two times?

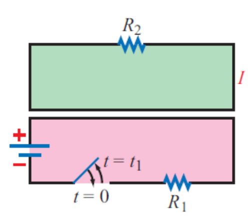

Figure P6.1: Loops of Problem 6.1.

Solution: The magnetic coupling will be strongest at the point where the wires of the two loops come closest. When the switch is closed the current in the bottom loop will start to flow clockwise, which is from left to right in the top portion of the bottom loop. To oppose this change, a current will momentarily flow in the bottom of the top loop from right to left. Thus the current in the top loop is momentarily clockwise when the switch is closed. Similarly, when the switch is opened, the current in the top loop is momentarily counterclockwise.

Problem 6.6 The square loop shown in Fig. P6.6 is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos(2\pi \times 10^4 t) \quad \text{(A)}.$$

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.

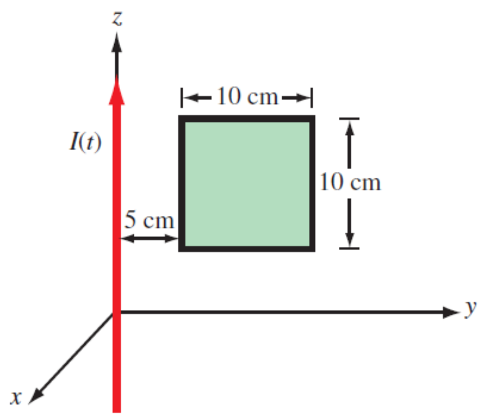

Figure P6.6: Loop coplanar with long wire (Problem 6.6).

Solution:

(a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\begin{aligned}
\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} &= \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y}\right) \cdot [-\hat{\mathbf{x}}\, 10 \text{ (cm)}]\, dy \\
&= \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln \frac{15}{5} \\
&= \frac{4\pi \times 10^{-7} \times 5\cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1 \\
&= 1.1 \times 10^{-7} \cos(2\pi \times 10^4 t) \quad \text{(Wb)}.
\end{aligned}$$

$$\begin{aligned}
V_{\text{emf}} = -\frac{d\Phi}{dt} &= 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7} \\
&= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \quad \text{(V)}.
\end{aligned}$$

(b)

$$I_{\text{ind}} = \frac{V_{\text{emf}}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \quad \text{(mA)}.$$

At $t = 0$, **B** is a maximum, it points in $-\hat{\mathbf{x}}$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be CCW when looking down on the loop, as shown in the figure.